CBCS SCHEME

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15MAT41

Fourth Semester B.E. Degree Examination, Jan./Feb. 2021
Engineering Mathematics - IV

Time: 3 hrs. Max. Marks: 80

Note: 1. ***Answer any FIVE full questions, choosing ONE full question from each module.***
2. ***Use of Statistical table is allowed.***

Module-1

1 a. Employ Taylor's Series Method to find 'y' at x = 0.2. Given the linear differential equation $\frac{dy}{dx} = 3e^x + 2y$ and y = 0 at x = 0 initially considering the terms upto the third degree. **(05 Marks)**

b. Use fourth order Runge – Kutta method to solve $(x + y)\frac{dy}{dx} = 1$, y(0.4) = 1 at x = 0.5 correct to four decimal places (Take h = 0.1). **(05 Marks)**

c. Apply Adams – Bash fourth method to solve $\frac{dy}{dx} = x^2(1 + y)$, given that y(1) = 1 , y(1.1) = 1.233 , y(1.2) = 1.548 and y(1.3) = 1.979 to evaluate y(1.4). **(06 Marks)**

OR

2 a. Given $\frac{dy}{dx} = x^2 + y$, y(0) = 1. Find correct to four decimal places y(0.1) using modified Euler's method taking h = 0.05. **(06 Marks)**

b. Use Milne's Predictor and Corrector method to compute y at x = 0.4, given $\frac{dy}{dx} = 2e^x - y$ and

x	0	0.1	0.2	0.3
y	2	2.010	2.040	2.090

(05 Marks)

c. Use Fourth order Runge – Kutta method to fond y(1.1), given $\frac{dy}{dx} + y - 2x = 0$, y(1) = 3 with step size h = 0.1. **(05 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} - x\frac{dy}{dx} - y = 0$ with the initial conditions y(0) = 1 , y'(0) = 0. Compute y(0.2) using Runge – Kutta method. **(05 Marks)**

b. Show that $J_{½}(1) = \sqrt{\frac{2}{\pi x}}\,\text{Sin } x$. **(05 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n$. **(06 Marks)**

OR

4 a. Apply Milne's method to compute y(0.8). Given that $\frac{d^2y}{dx^2} = 2y\frac{dy}{dx}$ and the following table of initial values. **(05 Marks)**

x	0	0.2	0.4	0.6
y	0	0.2027	0.4228	0.6841
y'	1	1.041	1.179	1.468

b. Express $f(x) = x^3 + 2x^2 - 4x + 5$ interms of Legendre Polynomials. **(05 Marks)**

c. Show that $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = 0$, If $\alpha \neq \beta$. Where α , β are roots of $J_n(x) = 0$. **(06 Marks)**

Module-3

5 a. Derive Cauchy – Riemann equations in Cartesian form. **(05 Marks)**

b. Using Cauchy's Residue theorem, evaluate the integral $\int_C \frac{ze^z}{z^2-1}dz$, where C is the circle $|Z| = 2$. **(05 Marks)**

c. Find the Bilinear transformation that transforms the points $Z_1 = 0$, $Z_2 = 1$, $Z_3 = \infty$ into the points $W_1 = -5$, $W_2 = -1$, $W_3 = 3$ respectively. **(06 Marks)**

OR

6 a. State and prove Cauchy's theorem. **(05 Marks)**

b. Evaluate $\int_C \frac{\sin^2 Z}{(Z-\pi/6)^3} dz$, where 'C' is the circle $|Z| = 1$, using Cauchy's integral formula. **(05 Marks)**

c. Construct the analytic function whose real part is $x + e^x \cos y$. **(06 Marks)**

Module-4

7 a. Obtain Mean and Variance of Exponential distribution. **(05 Marks)**

b. Find the binomial probability distribution which has mean 2 and variance $\frac{4}{3}$. **(05 Marks)**

c. The Joint probabilities distribution for two Random Variations X and Y as follows :

X \ Y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find i) Marginal distributions of X and Y ii) Co-variance of X and Y. Also verify that X and Y are independent iii) Correlation of X and Y. **(06 Marks)**

OR

8 a. A certain number of articles manufactured in one batch were classified into three categories according to a particular characteristic, being less than 50, between 50 and 60 and greater than 60. If this characteristic is known to be normally distributed, determine the mean and standard deviation for this batch if 60% , 35% and 5% were found in these categories. $[\phi(0.25) = 0.0987 , \phi(1.65) = 0.4505]$. **(05 Marks)**

b. Obtain the mean and standard deviation of Poisson distribution. **(05 Marks)**

c. Define Random variable. The pdf of a variate X is given by the following table :

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

i) Find K, if this represents a valid probability distribution.

ii) Find $P(x \geq 5)$ and $P(3 < x \leq 6)$. **(06 Marks)**

Module-5

9 a. Coins are tossed 100 times and the following results were obtained. Fit a binomial distribution for the data and test the goodness of fit [$\Psi^2_{0.05} = 9.49$ for 4 d.f]. **(06 Marks)**

Number of heads	0	1	2	3	4
Frequency	5	29	36	25	5

b. Find a Unique fixed Probability vector for the regular stochastic matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

(05 Marks)

c. A group of boys and girls were given an intelligence test. The mean score. S.D score and numbers in each group are as follows :

	Boys	Girls
Mean	74	70
SD	8	10
n	12	10

Is the difference between the means of the two groups significant at 5% level of significance [$t_{.05} = 2.086$ for 20 d,f]. **(05 Marks)**

OR

10 a. A coin was tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is unbiased at 5% level of significance. **(05 Marks)**

b. The weight of 1500 ball bearings are normally distributed with a mean of 635 gms and S.D of 1.36 gms. If 300 random samples of size 36 are drawn from this populations. Determine the expected mean and S.D of the sampling distribution of means if sampling is done i) With replacement ii) without replacement. **(05 Marks)**

c. Every year, a man trades his car for a new car. If he has a Maruti, he trades it for an Ambassador. If he has an Ambassador, he trades it for a Santro. However, if he has a Santro, he is just as likely to trade it for a new Santro as a trade it for a Maruti or an Ambassador. In 2000 he bought his first car which was a Santro. Find the probability that he has

i) 2002 Santro ii) 2002 Maruti. **(06 Marks)**

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